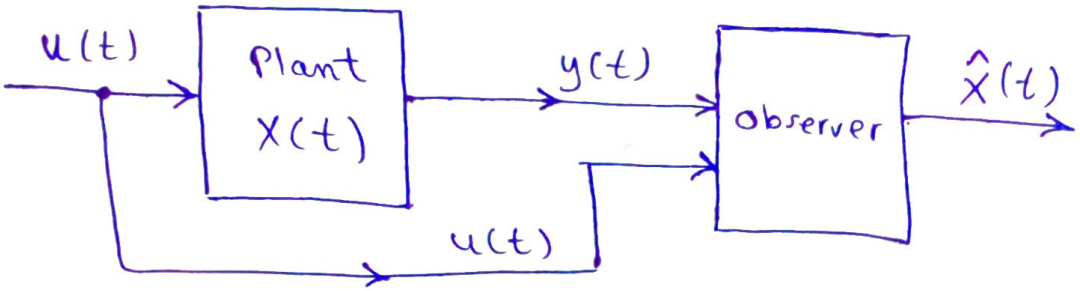
* Observer Design :-

the observer is used to estimate the unmeasurable states.

Observer

- full order observer. (The estimation is done for all states)
- reduced order observer. (The estimation is done for some states, not all states)



$$e(t) = X(t) - \hat{X}(t)$$

The observer equation:

$$\dot{\hat{X}}(t) = (A - GC)\hat{X}(t) + Bu(t) + Gy(t)$$

(G: gain matrix)

The design is to find the gain matrix G

to obtain the gain G, the design specs will be contained the required transient response for the observer (given desired poles)

[1] desired ch. equation $\alpha_0(s)$

$$\alpha_0(s) = (s - P_1)(s - P_2) \quad \text{---} \quad (1)$$

P_1, P_2 --- $\Rightarrow$ desired poles (given)

desired ch. equation in terms of G

$$|sI - (A - GC)| = 0 \quad (2)$$

2

$$G = \alpha_o(A)\, M_o^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

$M_o \Rightarrow$ observability matrix

$$\alpha_o(A) = \alpha_o(s) \Big|_{s \to A}$$

Ex: design an observer for the following system:-

$$\dot{X} = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} X(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & -1 \end{pmatrix} X(t)$$

the observer poles are located at $-1, -2$

① desired ch. eq.

$$\alpha_o(s) = (s+1)(s+2) = 0$$

$$\Rightarrow \boxed{\alpha_o(s) = s^2 + 3s + 2}$$

$$\Rightarrow \alpha_o(A) = A^2 + 3A + 2I$$

$$= \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}^2 + 3\begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 12 & 0 \\ 5 & 2 \end{pmatrix}$$

② $M_o = \begin{pmatrix} C \\ CA \end{pmatrix}$

$$CA = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$\Rightarrow M_o = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \Rightarrow M_o^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\therefore G = \alpha_o(A)\, M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\Rightarrow$ Turn Over

$$G = \begin{pmatrix} 12 & 0 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ - & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ 7 \end{pmatrix}$$